

CORRECTION DM N°1

Exercice 4.1.

1.

$$-\sqrt{9} = -3 \in \mathbb{Z} \quad ; \quad -\frac{23}{7} \in \mathbb{Q} \quad ; \quad \frac{12}{5} = 2.4 \in \mathbb{D}$$

$$\sqrt{2} - \frac{1}{\sqrt{2}-1} = \sqrt{2} - \frac{\sqrt{2}+1}{\sqrt{2}^2-1^2} = \sqrt{2} - (\sqrt{2}+1) = -1 \in \mathbb{Z}$$

2. (a) Entier naturel : 7824 (c) Entier non décimal : impossible (e) Rationnel non réel : impossible
 (b) Décimal non entier : 82.324 (d) Rationnel non entier : $\frac{983}{2}$ (f) Irrationnel : π

Exercice 4.2.

$$\begin{array}{llll} & (3x-2)(2x+1) = x(6x-2) & -\frac{6}{7}x + \frac{1}{7} = -\frac{3}{7} \\ & \Leftrightarrow 6x^2 + 3x - 4x - 2 = 6x^2 - 2x & \Leftrightarrow -\frac{6}{7}x = -\frac{4}{7} \\ (3x-8)(2x+1) = 0 & \Leftrightarrow -x-2 = -2x & \Leftrightarrow x = -\frac{4}{7} \times \frac{-7}{6} \\ \Leftrightarrow 3x-8 = 0 \text{ ou } 2x+1 = 0 & \Leftrightarrow x-2 = 0 & \Leftrightarrow x = \frac{2}{3} \\ \Leftrightarrow x = \frac{8}{3} \text{ ou } x = -\frac{1}{2} & \Leftrightarrow x = 2 & \Leftrightarrow x = \frac{2}{3} \\ S_{\mathbb{N}} = \emptyset \quad S_{\mathbb{Z}} = \emptyset & S_{\mathbb{N}} = \{2\} \quad S_{\mathbb{Z}} = \{2\} & S_{\mathbb{N}} = \emptyset \quad S_{\mathbb{Z}} = \emptyset \\ S_{\mathbb{Q}} = \left\{-\frac{1}{2}; \frac{8}{3}\right\} & S_{\mathbb{Q}} = \{2\} & S_{\mathbb{Q}} = \left\{\frac{2}{3}\right\} \end{array}$$

Exercice 4.3.

1. $5814 = 2 \times 3^2 \times 17 \times 19$ et $3876 = 2^3 \times 3^2 \times 53$
 2. $\sqrt{5814} = 3\sqrt{2 \times 17 \times 19}$ et $3816^5 = 2^{15} \times 3^{10} \times 53^5$
 3. $\text{pgcd}(5814^2, 3816^5) = \text{pgcd}(2^2 \times 3^4 \times 17^2 \times 19^2; 2^{15} \times 3^{10} \times 53^5) = 2^2 \times 3^4$
 $\text{ppcm}(5814; 3816) = 2^3 \times 3^2 \times 17 \times 19 \times 53$

Exercice 4.4.

1. $\frac{25^{-2} \times (15^3)^6}{9^4 \times (-5)^3} = -\frac{(5^2)^{-2} \times (3 \times 5)^{18}}{(3^2)^4 \times 5^3} = -\frac{5^{-4} \times 3^{18} \times 5^{18}}{3^8 \times 5^3} = -5^{-4-3+18} \times 3^{18-8} = -5^{11} \times 3^{10}$
 2. $A = (2x+1)^2 - (2x+1) = (2x+1)(2x+1-1) = (2x+1)(2x)$
 $B = (4x+3)(x+5) + 16x^2 + 24x + 9 = (4x+3)(x+5) + (4x+3)^2$
 $B = (4x+3)(x+5+4x+3) = (4x+3)(5x+8)$
 3. $A = 14\sqrt{50} - 8\sqrt{72} + \sqrt{2} + \sqrt{32} - 15 = 14 \times 5\sqrt{2} - 8 \times 3 \times 2\sqrt{2} + \sqrt{2} + 4\sqrt{2} - 15$
 $A = 70\sqrt{2} - 48\sqrt{2} + \sqrt{2} + 4\sqrt{2} - 15 = -15 + 17\sqrt{2}$

Exercise 4.5.

1. $\frac{1}{n} - \frac{1}{n+1} = \frac{n+1}{n(n+1)} - \frac{n}{n(n+1)} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}$

2.

$$\begin{aligned} S &= \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \frac{1}{8 \times 9} + \frac{1}{9 \times 10} \\ &= \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{1}{4} - \frac{1}{5} \right) + \left(\frac{1}{5} - \frac{1}{6} \right) + \left(\frac{1}{6} - \frac{1}{7} \right) \\ &\quad + \left(\frac{1}{7} - \frac{1}{8} \right) + \left(\frac{1}{8} - \frac{1}{9} \right) + \left(\frac{1}{9} - \frac{1}{10} \right) \\ &= 1 - \frac{1}{10} \\ &= \frac{9}{10} \end{aligned}$$

3.

$$\begin{aligned} S &= \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{(n-1) \times n} + \frac{1}{n \times (n+1)} \\ &= \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n} \right) + \left(\frac{1}{n} - \frac{1}{n+1} \right) \\ &= 1 - \frac{1}{n+1} \\ &= \frac{n+1-1}{n+1} \\ &= \frac{n}{n+1} \quad \text{CQFD} \end{aligned}$$